

METR 5223: Atmospheric Radiation

Elementary Adjustment
to Radiative Equilibrium
Part 2

Lecture for Spring 2009

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$$\tau \frac{dT'}{dt} + T' = T'_0$$

For notational simplicity, let:

$$x \equiv \frac{t}{\tau} \quad y \equiv T' \quad f \equiv T'_0$$

$$\frac{dy}{dx} + y = f(x)$$

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$$e^x \frac{dy}{dx} + e^x y = e^x f(x)$$

$$\frac{d}{dx} (e^x y) = e^x f(x)$$

$$e^x y|_{x_0}^x = \int_{x_0}^x f(x') e^{x'} dx'$$

$$e^x y(x) - e^{x_0} y(x_0) = \int_{x_0}^x f(x') e^{x'} dx'$$

$$y(x) = y(x_0)e^{x_0-x} + \int_{x_0}^x f(x')e^{x'-x} dx'$$

Define a **propagator**:

$$g(x', x) \equiv e^{x'-x}$$

Or, since $x > x'$, and to emphasize $g(x', x)$ represents exponential decay,

$$g(x', x) = e^{-(x-x')}$$

$$y(x) = y(x_0)g(x_0, x) + \int_{x_0}^x f(x')g(x', x) dx'$$

Note that x' is “more variable” than x .

Meaning when seeking a solution for y at x , x' is the “variable variable”.

$$g(x', x) \equiv e^{x'-x}$$

$$dg(x', x) = de^{x'-x} = e^{x'-x} dx' = g(x', x) dx'$$

So, because the propagator is exponential,

$$g(x', x)dx' = dg(x', x)$$

the solution converts to:

$$y(x) = y(x_0)g(x_0, x) + \int_{x_0}^x f(x')g(x', x)dx'$$

$$y(x) = y(x_0)g(x_0, x) + \int_{x_0}^x f(x')dg(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0, x)}^{g(x, x)} f(x')dg(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0, x)}^1 f(x')dg(x', x)$$

Consider a $f(x)$ that is represent as piece-wise constant:

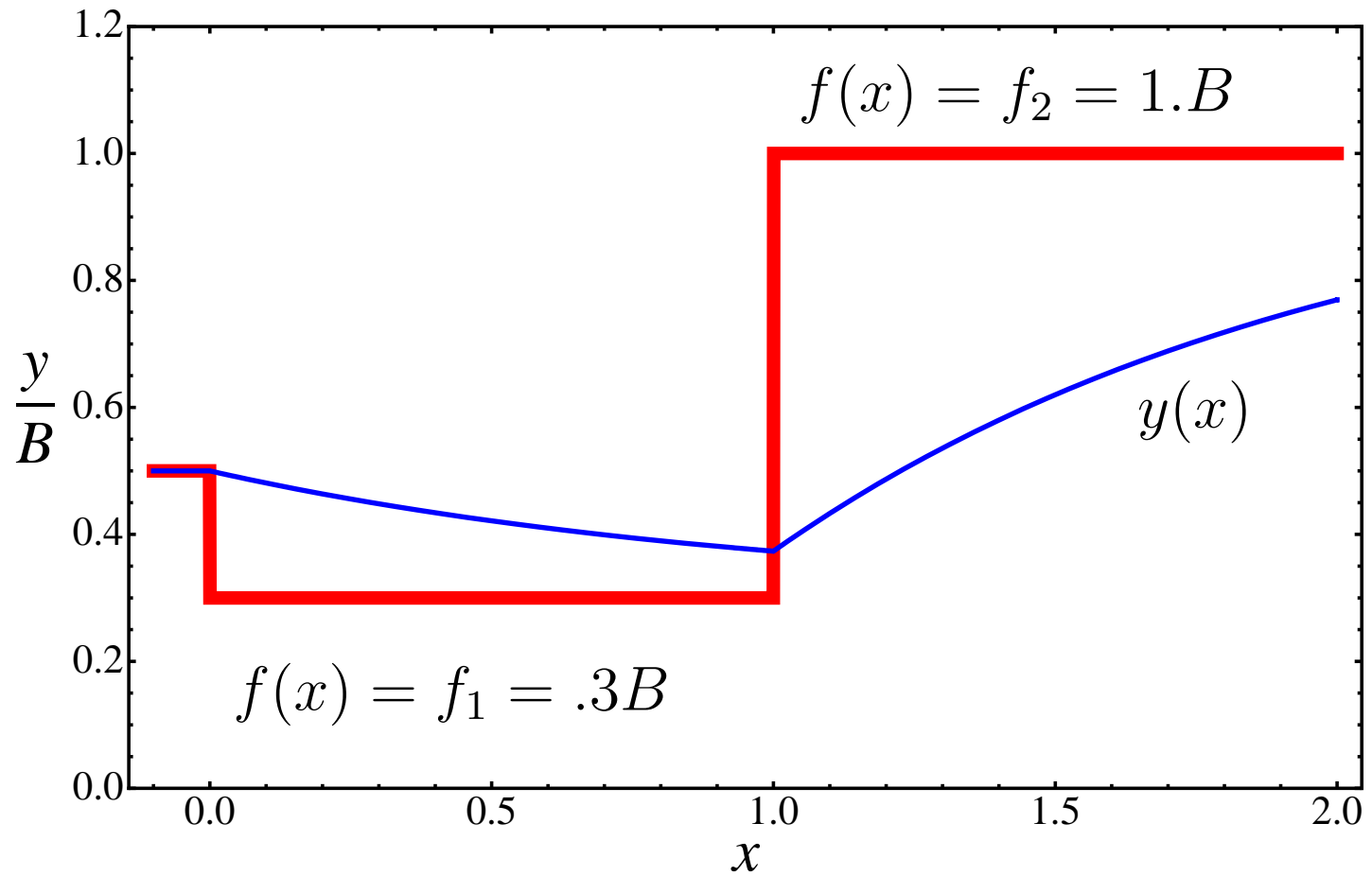
$$f(x) = f_1, \quad x_0 < x < x_1$$

$$f(x) = f_2, \quad x_1 < x < x_2$$

and so on. When solving for x in the second layer, or $x_1 < x < x_2$:

$$y(x) = y(x_0)g(x_0, x) + \int_{g(x_0, x)}^{g(x_1, x)} f_1 dg(x', x) + \int_{g(x_1, x)}^1 f_2 dg(x', x)$$

$$y(x) = y(x_0)g(x_0, x) + f_1 [g(x_1, x) - g(x_0, x)] + f_2 [1 - g(x_1, x)]$$



$y(x)$ for $y(0) = .5B$ and the depicted $f(x)$.

Reality check.

Suppose $f_1 = f_2 = y(x_0)$:

$$\begin{aligned}y(x) &= y(x_0)g(x_0, x) \\ &+ y(x_0) [g(x_1, x) - g(x_0, x)] \\ &+ y(x_0) [1 - g(x_1, x)] \\ &= y(x_0)\end{aligned}$$

Reality checks will help prevent certain common errors.

Here is how an error develops. This is **okay**:

$$\begin{aligned}y(x_2) = & \quad y(x_0)g(x_0, x_2) \\ & + f_1 [g(x_1, x) - g(x_0, x_2)] \\ & + f_2 [1 - g(x_1, x_2)]\end{aligned}$$

Adding another layer and simply recycling your previous computations is **not okay**:

$$\begin{aligned}y(x_3) = & \quad y(x_0)g(x_0, x_2) \\ & + f_1 [g(x_1, x) - g(x_0, x_2)] \\ & + f_2 [1 - g(x_1, x_2)] \\ & + f_3 [1 - g(x_2, x_3)]\end{aligned}$$