METR 5223: Atmospheric Radiation

Climate Sensitivity and Stability in a Simple Model

Lecture for Spring 2009

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Recall in the slab atmosphere model the irradiance or flux density out of the top of the atmosphere is:

$$F \uparrow = (1 - \epsilon)\sigma T_s^4 + \epsilon\sigma T_a^4$$

Recall we found that when the atmosphere is in equilibrium with the surface:

$$T_a^4 = \frac{1}{2} T_s^4$$

This equilibrium is quick, compared to the oceans coming into equilibrium with a change in solar radiation. Assuming the atmosphere is in equilibrium with the oceans:

$$F \uparrow = \left(1 - \frac{\epsilon}{2}\right) \sigma T_s^4$$

For an "aqua-planet" with a well-mixed ocean and a slab atmosphere:

$$C\frac{dT_s}{dt} = \frac{S_0}{4}(1-\alpha_p) - \left(1-\frac{\epsilon}{2}\right)\sigma T_s^4$$

We simplify notation with $T_s = T$ and $(1-\alpha_p) = B$ (think B for **blackness**) and $R = \left(1-\frac{\epsilon}{2}\right)$ (the greenhouse **reduction** factor) and $s = \frac{S_0}{4}$:

$$C\frac{dT}{dt} = sB - R\sigma T^4$$

In equilibrium:

$$R\sigma T^4 = sB$$

Use calculus to examine the sensitivity of the equilibrium solutions for T.

$$\ln\left(R\sigma T^4\right) = \ln\left(sB\right)$$

$$\ln(R) + \ln(\sigma) + 4\ln(T) = \ln(s) + \ln(B)$$

$$\frac{dR}{R} + 4\frac{dT}{T} = \frac{ds}{s} + \frac{dB}{B}$$

Recall the familiar case with B and R constant:

$$\frac{dT}{T} = \frac{1}{4}\frac{ds}{s}$$

Now investigate feedback. Suppose $B = B(T)$, but the changes in R and S are imposed, and independent of T:

$$\frac{dR}{R} + 4\frac{dT}{T} = \frac{ds}{s} + \frac{1}{B}\frac{dB}{dT}dT$$

$$4\frac{dT}{T} - \frac{1}{B}\frac{dB}{dT}dT = \frac{dP}{P}$$

where P can be called the **punch**

$$P \equiv \frac{s}{R}$$

$$\frac{dT}{T}\left(1 - \frac{1}{4}\frac{T}{B}\frac{dB}{dT}\right) = \frac{1}{4}\frac{dP}{P}$$

Increasing s or decreasing R give postive changes in P.

$$\frac{dT}{dP} = G\frac{1}{4}\frac{T}{P}$$

where the **gain** G is:

$$G \equiv \frac{1}{1-f}$$

and f is the **feedback** factor:

$$f \equiv \frac{1}{4}\frac{d\ln B}{d\ln T}$$

When $f = 0$, $G = 1$. We can write:

$$\frac{dT}{dP} = G\left(\frac{dT}{dP}\right)_{\text{no-feedback}}$$

$$G = \frac{1}{1-f}$$

- $0 < G < 1$ for $f < 0$. This is negative feedback.
- $G > 1$ for $0 < f < 1$. This is positive feedback.
- $G < 0$ for $f > 1$. Unstable equilibrium point.

Attempt to estimate dB/dT for Earth and f for Albedo feedback.

Suppose the temperature drops by 30 K. Earth becomes frozen and completely white. B goes from 0.7 to 0.

$$\frac{dB}{dT} \approx \frac{\Delta B}{\Delta T} = \frac{-0.7}{-30 \text{ K}} = .023 \text{ K}^{-1}$$

$$f = \frac{1}{4}\frac{T}{B}\frac{dB}{dT} = \frac{1}{4}\frac{300 \text{ K}}{0.7}\frac{0.7}{30 \text{ K}} = 2.5$$

$f = 2.5$ is an overestimate by *at least a factor of 10* for current polar ice:

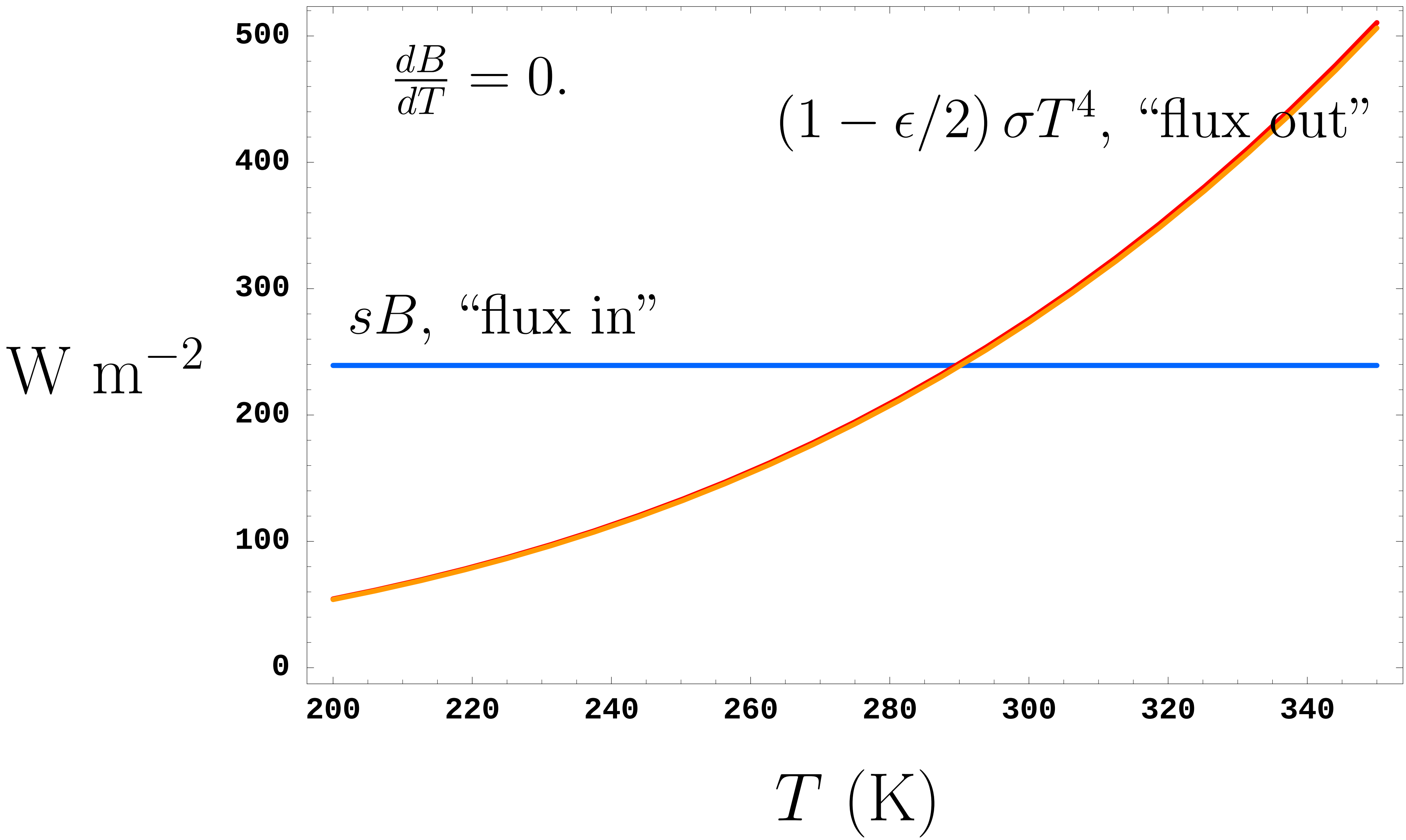
1. Ice is not perfectly reflective, esp. in the near IR.
2. High latitudes have small surface area.
3. At large zenith angle water is highly reflective.

For an example of moderate ice-albedo feedback we could use:

$$f = 0.25 \qquad G = \frac{1}{1-f} = 1.33$$

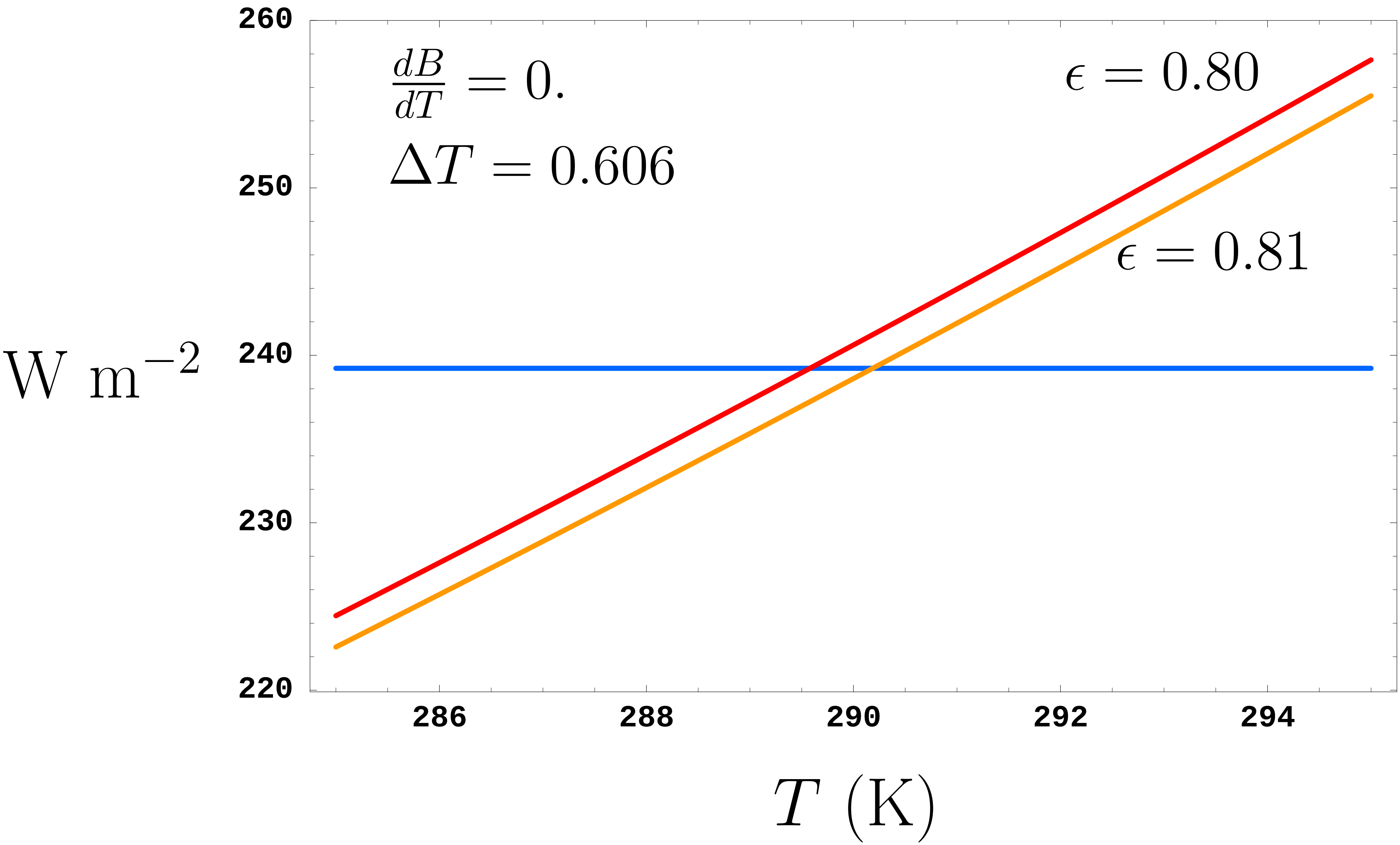
Turn now to a graphical presentation of sensitivity and feedback...

B constant, no feedback:



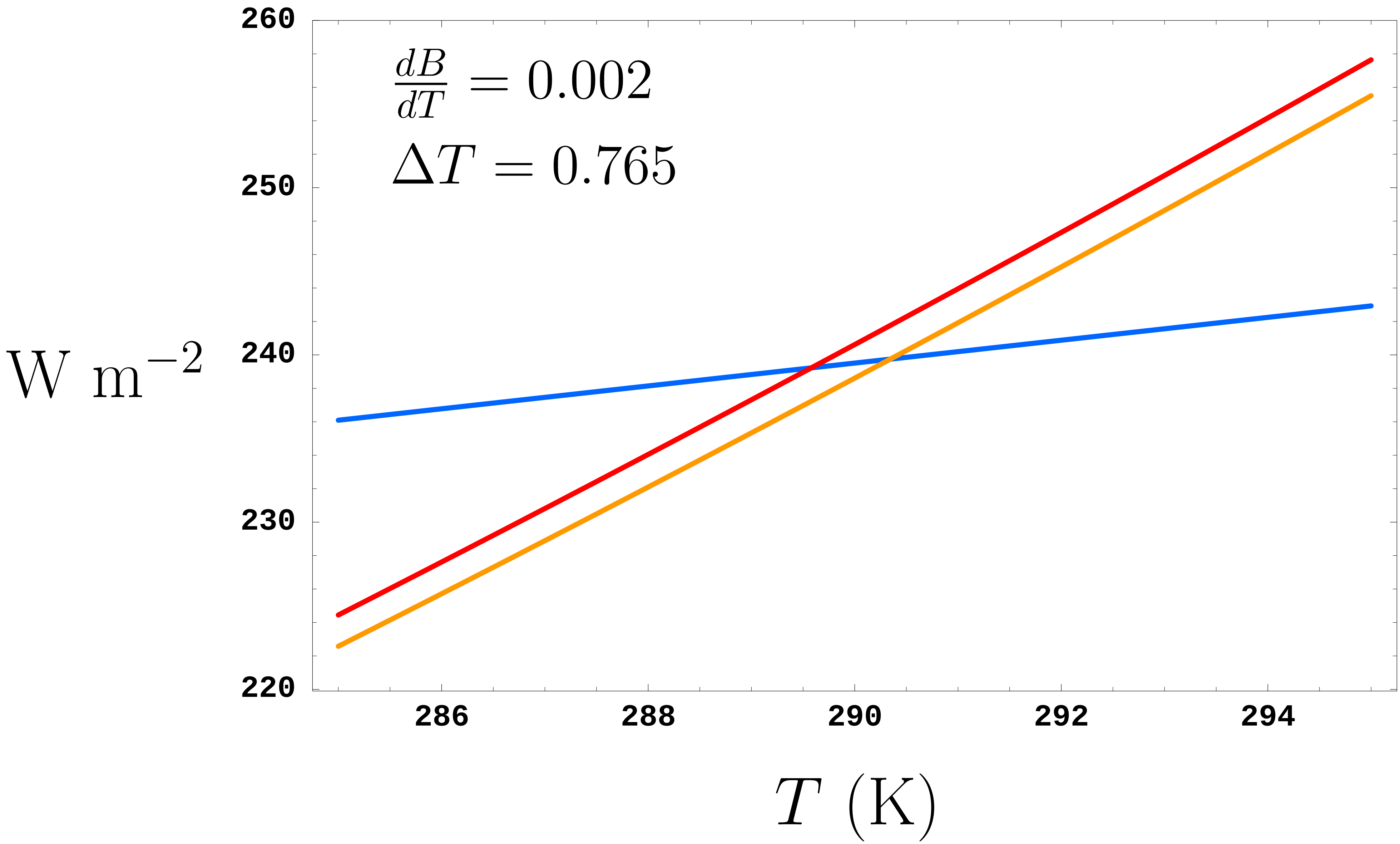
"flux out" curves for $\epsilon = .80$ and $\epsilon = .81$ nearly overlap

B constant, no feedback, zoom:



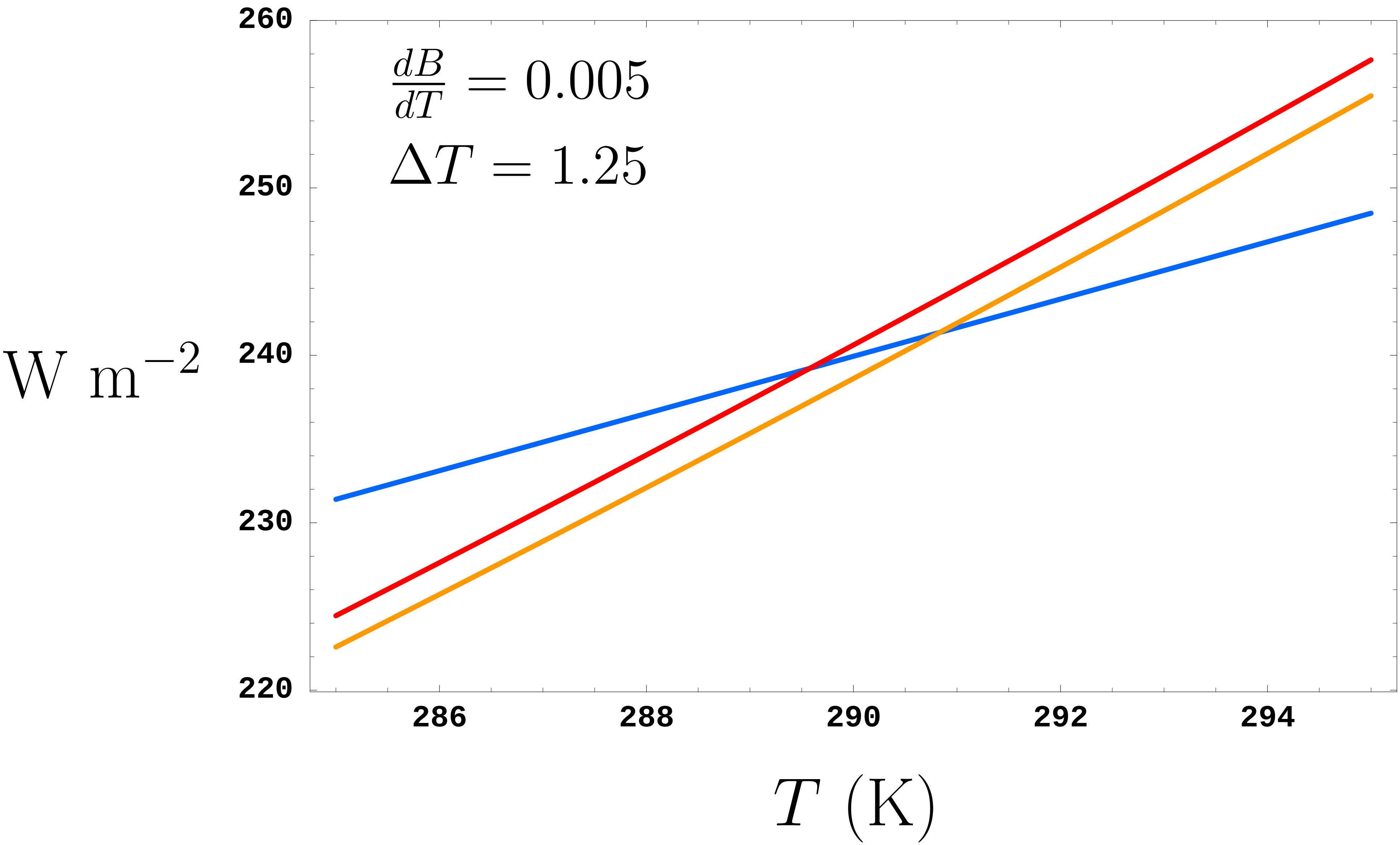
As $\epsilon = .80 \rightarrow \epsilon = .81$, T increases by $\Delta T = 0.606$

B increases with T, positive feedback :



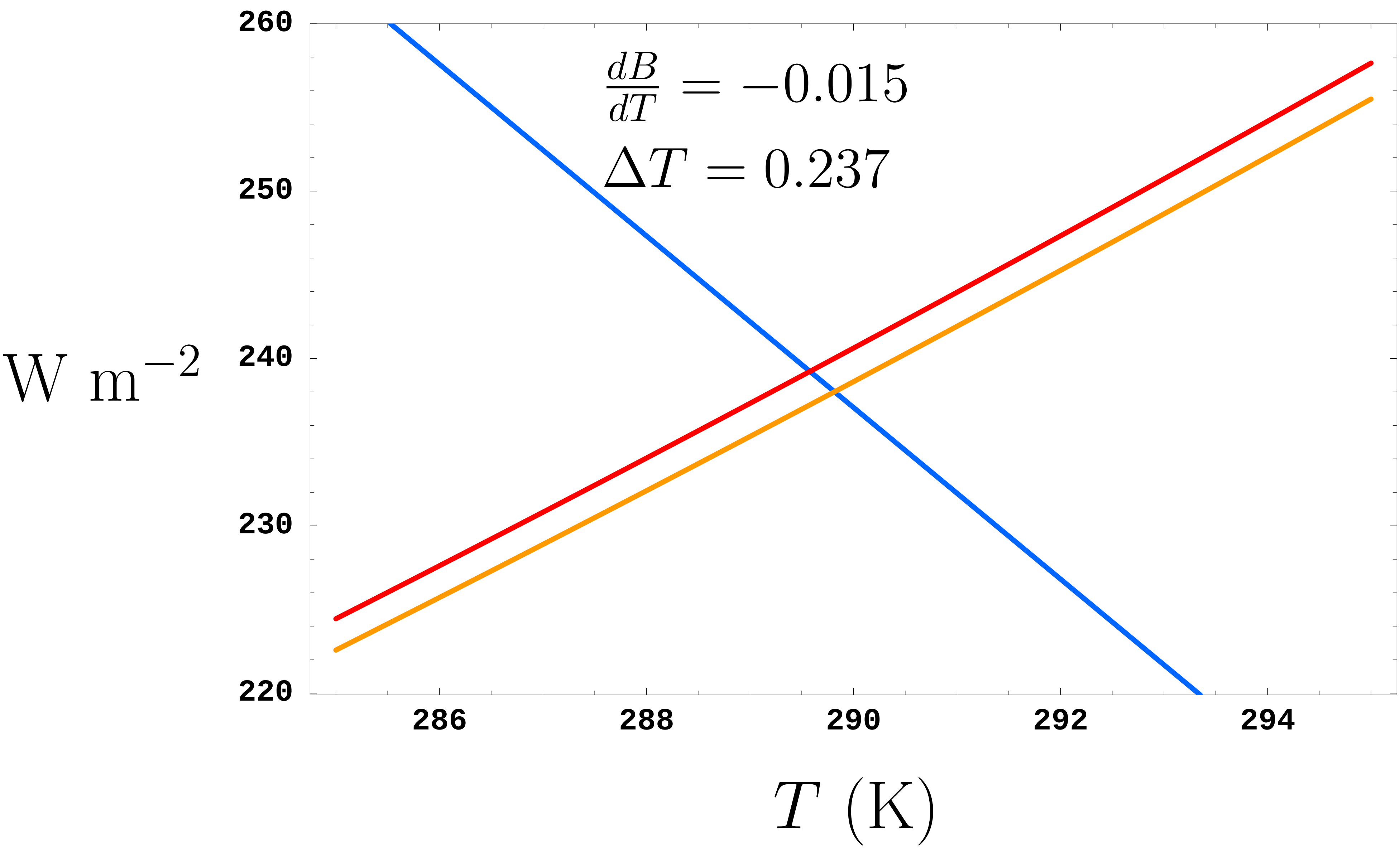
$\Delta T = 0.785$, no-feedback was $\Delta T = 0.606$

larger positive feedback :



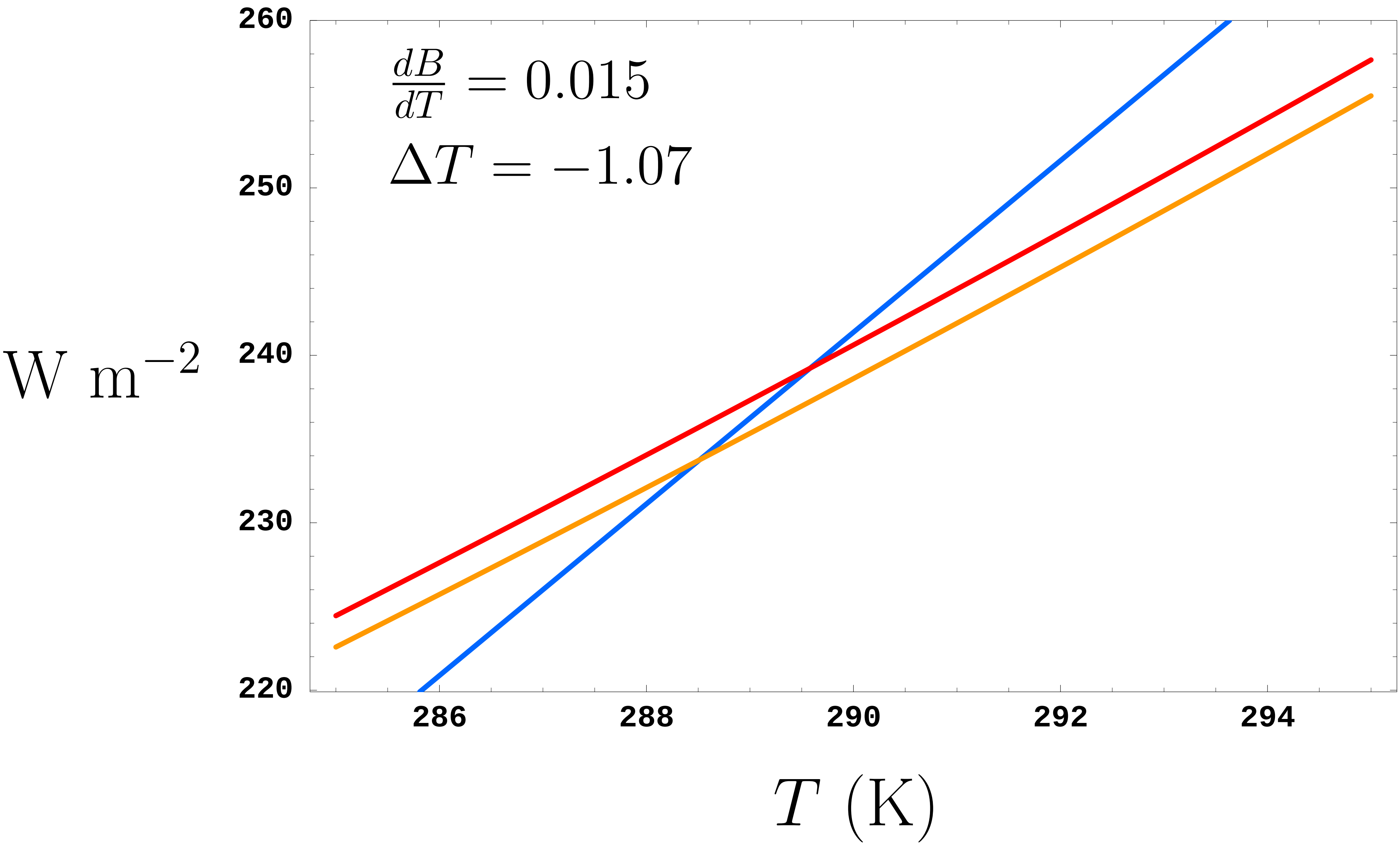
The gain $G = 2.1$

B decreases with T, negative feedback:



$\Delta T > 0$, but less than no-feedback case of $\Delta T = 0.606$

very large positive feedback, equilibrium point unstable:



equilibrium analysis of the unstable point yields $\Delta T < 0$

summary: